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# THEORETICAL AND NUMERICAL ANALYSIS VIBRATION STUDY OF ISOTROPIC HYPER COMPOSITE PLATE STRUCTURAL

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## ABSTRACT

In this work, a suggested analytical solution for dynamic analysis of hyper composite plate combined from two reinforcement finer, mat and powder or short and powder reinforcement fiber, with resin matrix, as polyester or epoxy resin, is presented. The theoretical study of hyper composite plate evaluated the natural frequency of plate with different volume fraction of reinforcement fiber and resin matrix effect and evaluated the effect of reinforcement fiber and resin types on the natural frequency of plate. The a suggested analytical solution included evaluated the mechanical properties of isotropic hyper composite material plate, combine from powder reinforcement and mat or short reinforcement fiber with polyester or epoxy resin matrix, as modulus of elasticity and modulus of rigidity in addition to Poisson's ratio of hyper composite plate. And, solution for the general equation of motion for an isotropic hyper composite plate with effect of the volume fraction and types of reinforcement fiber and matrix resin.

The results shown the natural frequency increasing with increasing of reinforcement fiber and increasing with increasing of strength reinforcement fiber or resin matrix but the natural frequency of hyper composite plate non-effect with increasing or decreasing (change) of powder reinforcement of hyper composite plate. A comparison made between analytical results from theoretical solution of general equation of motion of hyper composite plate, with different volume fraction and types of reinforcement fiber and resin matrix effect, with numerical solution, by ANSYS program Ver. 14, results, given good agreement with maximum error about 1.8% and minimum error about 0.75%.

**KEYWORDS:** Vibration of Composite Plate, Mechanical Properties of Composite Materials, Hyper Composite Materials, Isotropic Composite Materials, Vibration Analysis of Isotropic Composite Plate

## INTRODUCTION

Composite material reveals a material that is different from common heterogeneous materials. Currently composite materials refers to materials having strong fibers-continuous or non-continuous-surrounded by a weaker matrix material. The matrix serves to distribute the fibers and also to transmit the load to the fibers. The bonding between fibers and matrix is created during the manufacturing phase of the composite material. This has fundamental influence on the mechanical properties of the composite material, **D. Gay et al. [1]**.

In composites, materials are combined in such a way as to enable us to make better use of their virtues while minimising to some extent the effects of their deficiencies. This process of optimisation can release a designer from the constraints associated with the selection and manufacture of conventional materials. He can make use of tougher and lighter materials, with properties that can be tailored to suit particular design requirements. And because of the ease with which complex shapes can be manufactured, the complete rethinking of an established design in terms of composites can often lead to both cheaper and better solutions. The simple term 'composites' gives little indication of the vast range of individual combinations that are included in this class of materials, **Bryan Harris [10]**.

Many studies were performed to examine the vibration study of different composite plate and reinforcement fiber and matrix resin types, as,

**R.C.L. Dutra et. al. [5]**, polypropylene fiber and mercapto-modified polypropylene blend fiber (PPEVASH) were combined with carbon fiber as reinforcing elements in so-called hybrid epoxy composites. The impact performance and dynamic mechanical properties of these materials were investigated. Hybrid composites containing PPEVASH blend fibers display higher impact strength than plain carbon fiber composites but the performance is lower than plain PPEVASH-epoxy composite.

**K.K. Shukla et. al. [7],** the paper presents an analytical approach to examine the nonlinear dynamic responses of a laminated composite plate composed of spatially oriented short fibers in each layer of the composite. Using Mori–Tanaka mean field theory, the effective elastic moduli of each lamina are obtained explicitly as a function of the properties of the constituents, volume fraction, orientation angles, and fiber shape.

**Ali M. H. Y. Al-Hajjar [9]**, an analytical and numerical study of simply supported unidirectional hyper composite plate is presented in this paper. The composite plate is composed of powder glass reinforcement in polyster resin as a matrix and long glass fibers as reinforcement. The analytical method used in this paper to find the general equation of motion is the specially orthotropic plate equations.

**Gururaja M N and A N Hari Rao [11]**, this paper presents a review of the current status of hybrid composite materials technology, in terms of materials available and properties, and an outline of some of the trends, obvious and speculative, with emphasis on various applications including some details of smart hybrid composites.

In this research a suggested analytical solution of vibration study of hyper composite plate, combined from two reinforcement fiber types and resin matrix, to evaluated the natural frequency of hyper composite plate with different volume fraction of each types of reinforcement fiber and resin matrix. And camper the theoretical results of natural frequency with numerical results evaluated by finite element method, by using of ANSYS program Ver. 14.

## MECHANICAL PROPERTIES OF HYPER COMPOSITE MATERIAL PLATE STRUCTRAL

### Mechanical Properties of Composite Matrix, (Resin and Powder Reinforcement Fiber)

Spherical fillers are reinforcements associated with polymer matrices. They are in the form of micro-balls, either solid or hollow, with diameters between 10 and 150 μm. They are made of glass, carbon, or polystyrene. The composite (matrix + filler) is isotropic, with elastic properties E, G, ν given by the following relations, **D. Gay et al. [1]**,

$$K = \frac{E_m}{3.(1-2.\nu_m)}\left[1 + 3.\left(\frac{1-\nu_m}{1+\nu_m}\right).\frac{\forall_{pf}}{\forall_m}\right], E \approx \frac{9.K.G}{3.K+G}, G = \frac{E_m}{2.(1+\nu_m)}.\left[1 + \frac{15}{2}.\left(\frac{1-\nu_m}{4-5.\nu_m}\right).\frac{\forall_{pf}}{\forall_m}\right], \nu = \frac{1}{2}.\left(\frac{3.K-2.G}{3.K+G}\right) \quad (1)$$

Where, $E_m$ modulus of elasticity of resin, $\nu_m$ Poisson's ratio of resin, $\forall_{pf}$ volume fraction of reinforcement powder, and $\forall_m$ volume fraction of resin.

Then, the mechanical properties of composite matrix combined from (resin and powder reinforcement fiber) are,

$$E_{cm} = E,\ G_{cm} = G,\ \nu_{cm} = \nu \quad (2)$$

And, volume fraction of composite matrix combined from resin and powder reinforcement is,

$$\forall_{cm} = \forall_{pf} + \forall_m \quad (3)$$

**Mechanical Properties of Hyper Composite Materials Plate, (Resin Matrix, Powder and Mat or Short Fiber)**

The properties of hyper composite material plate calculated of two types of reinforcement fiber, mat reinforcement fiber and short reinforcement fiber, as,

- **Mechanical Properties of Mat Hyper Composite Materials**

Mats are made of cut fibers (fiber lengths between 5 and 10 cm) or of continuous fibers making a bidimensional layer. Mats are isotropic within their plane (x, y). If $E_1$ and $E_2$ are the elastic module (along the longitudinal and transverse directions) of an unidirectional ply with the same volume fraction of $\forall_f$, one has, **D. Gay et al. [1]**,

$$E_{mat} = \frac{3}{8}E_1 + \frac{5}{8}E_2,\quad G_{mat} \approx \frac{E_{mat}}{2.(1+\nu_{mat})},\quad \nu_{mat} \approx 0.3 \tag{4}$$

Where, $E_1$ and $E_2$ are defined as, **D. Gay et al. [1]**,

$$E_1 = E_{m_f}.\forall_{m_f} + E_{cm}.\left(1-\forall_{m_f}\right),\ E_2 = \frac{E_{cm}}{\left(1-\forall_{m_f}\right)+\frac{E_{cm}}{E_{m_f}}\forall_{m_f}} \tag{5}$$

Where, $E_{m_f}$ modulus of elasticity of mat reinforcement, and $\forall_{m_f}$ volume fraction of mat reinforcement.

And, the density of hyper composite plate combined from (resin, powder and mat reinforcement fiber) is,

$$\rho = \rho_{m_f}.\forall_{m_f} + \rho_m.\forall_m + \rho_{p_f}.\forall_{p_f} \tag{6}$$

Where, $\rho_{m_f}$, $\rho_m$, $\rho_{p_f}$ are density of resin and reinforcement mat and powder fiber, respectively.

- **Mechanical Properties of Short Hyper Composite Materials**

For unidirectional fiber matrix shown in **Figure 1a** the following Halpin-Tsai relations are used to determine the elastic properties, **J. S. Rao [2]**,

$$E_{1m} = \frac{1+2.a_f.\eta_l.\forall_{sf}}{1-\eta_l.\forall_{sf}}.E_{cm},\quad E_{2m} = \frac{1+2.\eta_T.\forall_{sf}}{1-\eta_T.\forall_{sf}}.E_{cm}$$

$$G_{12m} = G_{21m} = \frac{1+\eta_G.\forall_{sf}}{1-\eta_G.\forall_{sf}}.G_{cm},\quad \nu_{12m} = \nu_{sf}.\forall_{sf} + \nu_{cm}.\forall_{cm} \tag{7}$$

Where, $a_f = 100$, and,

$$\eta_l = \frac{\left(\frac{E_{sf}}{E_{cm}}-1\right)}{\left(\frac{E_{sf}}{E_{cm}}+2.a_f\right)} = \frac{\left(E_{sf}-E_{cm}\right)}{\left(E_{sf}+2.a_f.E_{cm}\right)},\quad \eta_T = \frac{\left(\frac{E_{sf}}{E_{cm}}-1\right)}{\left(\frac{E_{sf}}{E_{cm}}+2\right)} = \frac{\left(E_{sf}-E_{cm}\right)}{\left(E_{sf}+2.E_{cm}\right)},\quad \eta_G = \frac{\left(\frac{G_{sf}}{G_{cm}}-1\right)}{\left(\frac{G_{sf}}{G_{cm}}+1\right)} = \frac{\left(G_{sf}-G_{cm}\right)}{\left(G_{sf}+G_{cm}\right)}$$

And, $E_{sf}$, $G_{sf}$, $\nu_{sf}$ are mechanical properties of short finer, $\forall_{sf}$ volume fraction of short fiber.

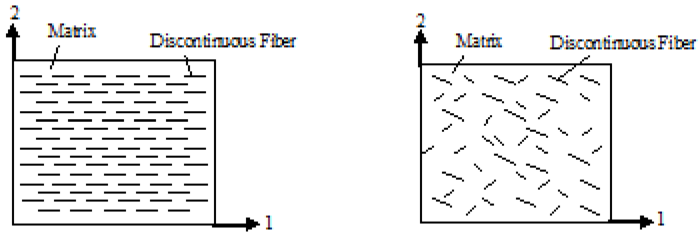
Let $E_{1m}$ and $E_{2m}$ be the longitudinal and transverse module defined by Eq. 7. for a unidirectional discontinuous fiber $0^0$ composite matrix of the same fiber aspect ratio and fiber volume fraction as the randomly oriented discontinuous fiber matrix shown in **Figure 1b**. Since the fiber is randomly oriented, the matrix exhibits isotropic behavior. The Young's modulus and shear modulus of such a composite matrix are given by, **J. S. Rao [2]**,

$$E_{short} = \frac{3}{8}E_{1m} + \frac{5}{8}E_{2m},\quad G_{short} = \frac{1}{8}E_{1m} + \frac{1}{4}E_{2m},\quad \nu_{short} = \left(\frac{E_{short}}{2.G_{short}}-1\right) \tag{8}$$

And the density of hyper composite plate combined from (resin matrix and power and short fiber) is as,

$$\rho = \rho_{s_f} . \forall_{s_f} + \rho_m . \forall_m + \rho_{p_f} . \forall_{p_f} \tag{9}$$

Where, $\rho_{s_f}$ density of short reinforcement finer.




**a- Unidirectional Discontinuous Matrix** **b- Randomly Oriented Discontinuous Fiber Matrix**

**Figure 1: Short Fiber Composite Materials, J. S. Rao [2]**

## A SUGGESTED VIBRATION ANALYSIS STUDY OF HYPER COMPOSITE PLATE (SPECIAL ORTHOTROPIC PLATE EQUATION)

We can determine the expresions for the bending and twisting moments with the displacement and strain fields as in the following equations, **J. S. Rao [2],**

$$U_x = -zw_{,x},\ U_y = -zw_{,y},\ U_z = w \tag{10}$$

And, the strain field of plate defined as**, J. S. Rao [2]**,

$$\varepsilon_{xx} = -z\, w_{,xx},\ \varepsilon_{yy} = -z\, w_{,yy},\ \gamma_{xy} = -2z\, w_{,xy} \tag{11}$$

The stresses can be written as the following, **J. S. Rao [2]**,

$$\sigma_{xx} = \frac{E_{xx}}{1 - v_{xy} v_{yx}} \varepsilon_{xx} + \frac{v_{xy} E_{yy}}{1 - v_{xy} v_{yx}} \varepsilon_{yy}$$

$$\sigma_{yy} = \frac{v_{xy} E_{yy}}{1 - v_{xy} v_{yx}} \varepsilon_{xx} + \frac{E_{yy}}{1 - v_{xy} v_{yx}} \varepsilon_{yy}$$

$$\tau_{xy} = G_{xy} \gamma_{xy} \tag{12}$$

Where, $E_{xx} = E_{yy} = E - for\ Isotropic\ Plate\ Materials$ , $G_{xy} = G - for\ Isotropic\ Plate\ Materials$ , $v_{xy} = v_{yx} = v - for\ Isotropic\ Plate\ Materials$.

Substituting for strain equations (11) in (12) to get,

$$\sigma_{xx} = -z \left( \frac{E_{xx}}{1 - v_{xy} v_{yx}} w_{,xx} + \frac{v_{xy} E_{yy}}{1 - v_{xy} v_{yx}} w_{,yy} \right)$$

$$\sigma_{yy} = -z \left( \frac{v_{xy} E_{yy}}{1 - v_{xy} v_{yx}} w_{,xx} + \frac{E_{yy}}{1 - v_{xy} v_{yx}} w_{,yy} \right)$$

$$\tau_{xy} = -2G_{xy}\, z\, w_{,xy} \tag{13}$$

The bending moments (per unit length) $M_x$, $M_y$ and $M_{xy}$ are then determined as, **J. S. Rao [2]**,

$$M_x = \int_{-h/2}^{h/2} \sigma_{xx} z\, dz = -\int_{-h/2}^{h/2} z^2 \left( \frac{E_{xx}}{1-v_{xy}v_{yx}} w_{,xx} + \frac{v_{xy}E_{yy}}{1-v_{xy}v_{yx}} w_{,yy} \right) dz = -\left( D_{11}w_{,xx} + D_{12}w_{,yy} \right)$$

$$M_y = \int_{-h/2}^{h/2} \tau_{yy} z\, dz = -\int_{-h/2}^{h/2} z^2 \left( \frac{v_{xy}E_{yy}}{1-v_{xy}v_{yx}} w_{,xx} + \frac{E_{yy}}{1-v_{xy}v_{yx}} w_{,yy} \right) dz = -\left( D_{22}w_{,yy} + D_{12}w_{,xx} \right)$$

$$M_{xy} = \int_{-h/2}^{h/2} \tau_{xy}\, z\, dz = -\int_{-h/2}^{h/2} 2G_{xy} z^2 w_{,xy}\, dz = 2D_{66}w_{,xy} \tag{14}$$

Where, in the particular case of isotropic in xy plane,

$$D_{11} = D_{22} = \frac{Eh^3}{12(1-\nu^2)},\ D_{12} = \frac{\nu Eh^3}{12(1-\nu^2)},\ D_{66} = \frac{Gh^3}{12} \tag{15}$$

The differential equation for plate is, **J. S. Rao [2]**,

$$M_{x,xx} - 2M_{xy,xy} + M_{y,yy} = -q \tag{16}$$

Substituting for the bending and twisting moments from equation (14) into eq. 16. So the above equations will be,

$$-\left(D_{11}w_{,xx} + D_{12}w_{,yy}\right)_{,xx} - 4\left(D_{66}w_{,xy}\right)_{,} - \left(D_{22}w_{,yy} + D_{12}w_{,xx}\right)_{,yy} = -q$$

Or,

$$D_{11}w_{,xxxx} + 2(D_{12} + 2D_{66})w_{,xxyy} + D_{22}w_{,yyyy} = q \tag{17}$$

For a vibration plate, the diffrential equation 17 is modified as, **J. S. Rao [2]**,

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} + \rho h\ \frac{\partial^2 w}{\partial t^2}\ddot{w} = q(x,y,t) \tag{18}$$

For free vibration, equation 18 is,

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} + \rho h\ \frac{\partial^2 w}{\partial t^2}\ddot{w} = 0 \tag{19}$$

Consider a simply supported rectangular plate of width a in x-direction and length b in y-direction with thickness h acted on by a static force q(x,y). The diffrerential equation for this problem is given by equation 17 and boundary conditions as, **J. S. Rao [2]**,

$$M_x = -\left(D_{11}w_{,xx} + D_{12}w_{,yy}\right) = 0, w = 0, on\ the\ edges\ x = 0\ and\ x = a$$

$$M_y = -\left(D_{22}w_{,yy} + D_{12}w_{,xx}\right) = 0, w = 0, on\ the\ edges\ y = 0\ and\ y = b \tag{20}$$

The solution of equation (19) satisfying the boundary conditions equation (20) can be written as, **J. S. Rao [2]**,

$$w = A\, sin\frac{m\pi x}{a} sin\frac{n\pi y}{b} sin\omega t \tag{21}$$

Where *m* and *n* are integers. Substitute the above and its derivatives in equation (19) to get,

$$D_{11}\left(\frac{m\pi}{a}\right)^4 + 2(D_{12} + 2D_{66})m^2n^2\frac{\pi^4}{a^2b^2} + D_{22}\left(\frac{n\pi}{b}\right)^4 = \rho h^2\omega_{mn}^2 \tag{22}$$

Therefor the natural frequency equation will be,

$$\omega_{mn}^2 = \frac{\pi^4}{\rho h a^4}[D_{11}m^4 + 2(D_{12} + 2D_{66})m^2n^2k^2 + D_{22}n^4k^4] \tag{23}$$

For, $k = \frac{a}{b}$, And, $D_{11} = \frac{Eh^3}{12(1-\nu^2)}$, $D_{22} = \frac{Eh^3}{12(1-\nu^2)}$, $D_{12} = \frac{\nu E h^3}{12(1-\nu^2)}$, $D_{66} = \frac{Gh^3}{12}$ – for isotropic plate.

Where, $E = E_{mat}$ – For, mat reinforcement fiber, and $E_{short}$ – for short reinforcement fiber.

$G = G_{mat}$ – For, mat reinforcement fiber, and $G_{short}$ – for short reinforcement fiber.

$\nu = \nu_{mat}$ – For, mat reinforcement fiber, and $\nu_{short}$ – for short reinforcement fiber.

## NUMERICAL ANALYSIS STUDY

The numerical study of natural frequency of hyper composite plate evaluated by using the finite elements method was applied by using the ANSYS program (ver.14). The three dimensional model were built and the element (Solid Tet 10 node 187) were used. Solid 187 elements is a higher order 3-D, 10-node element. Solid 187 has a quadratic displacement behaviour and is well suited to modelling irregular meshes. The element is defined by 10 nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. The element has plasticity, hyper-elasticity, creep, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials, and fully incompressible hyper-elastic materials. In addition to the nodes, the element input data includes the orthotropic or anisotropic material properties. Orthotropic and anisotropic material directions correspond to the element coordinate directions. The geometry, node locations, and the coordinate system for this element are shown in Figure 2. A sample of meshed plate with different aspect ratio is shown in Figure 3.

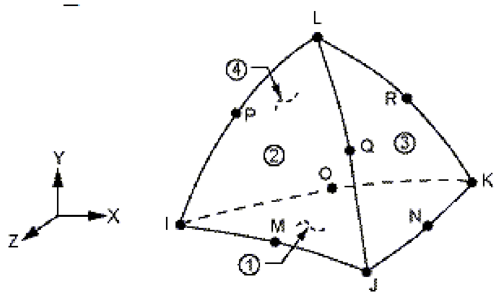
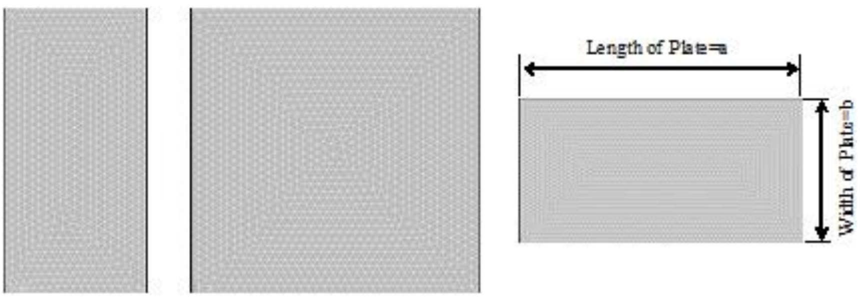



**Figure 2: Geometry of Solid 187 Element**




**a. Aspect Ratio=AR=0.5 b. Aspect Ratio=AR=1 c. Aspect Ratio=2**

**Figure 3: Meshed of Hyper Composite Plate with Different Aspect Ratio, with ANSYS Program**

## RESULTS AND DISCUSSIONS

The results of isotropic hyper composite plate studied, included evaluated the mechanical properties and natural frequency of composite plate structure combined from different reinforcement (glass of mat and short reinforcement fiber,

and, glass and boron powder reinforcement) and resin matrix materials types (polyester and epoxy resin materials) with different volume fraction of reinforcement and resin matrix. And compare the theoretical results of natural frequency of composite plate with numerical results evaluated by finite elements method, by using Ansys program ver. 14.

The combine types of isotropic hyper composite plate structures studies are,

- Glass Mat and Powder Reinforcements Fiber and Polyester Resin.
- Glass Mat and Boron Powder Reinforcements Fiber and Polyester Resin
- Glass Mat and Powder Reinforcements Fiber and Epoxy Resin
- Glass Mat and Boron Powder Reinforcements Fiber and Epoxy Resin
- Glass Short and Powder Reinforcements Fiber and Polyester Resin
- Glass Short and Boron Powder Reinforcements Fiber and Polyester Resin
- Glass Short and Powder Reinforcements Fiber and Epoxy Resin
- Glass Short and Boron Powder Reinforcements Fiber and Epoxy Resin

Where, the mechanical properties and density of (glass and boron) reinforcement fiber materials (short, mat, and powder) and (polyester and epoxy) resin materials used for combined the different isotropic hyper composite plates types structural are shown in table 1, **D. Gay et al. [1]**.

**Table 1: Density of Glass Fiber and Polyester Resin Materials, D. Gay et al. [1]**

| Materials | ρ( kg/m3 ) | E (Gpa) | G (Gpa) | ν |
|---|---|---|---|---|
| Glass Fibers | **2600** | **74** | 30 | 0.25 |
| Boron Fiber | 2600 | 400 | / | / |
| Polyester | 1200 | 4 | 1.4 | 0.4 |
| Epoxy | 1200 | 4.5 | 1.6 | 0.4 |

And the dimensions (length (a), width (b), and thickness (h)) of simply supported isotropic hyper composite plate using to evaluated the natural frequency, with different aspect ratio (AR=0.5, 1, and 2) and volume fraction of reinforcement and resin materials, of different composite plate structure types, are,

For, $AR = \frac{a}{b} = 0.5, lenght = a = 12.5\ cm, widht = b = 25\ cm, thickness = h = 5\ mm$

For, $AR = \frac{a}{b} = 1, lenght = a = 25\ cm, widht = b = 25\ cm, thickness = h = 5\ mm$

For, $AR = \frac{a}{b} = 2, lenght = a = 50\ cm, widht = b = 25\ cm, thickness = h = 5\ mm$ (24)

The mechanical properties of hyper composite materials types are shown in Table 2. for different mat reinforcement fiber with different powder reinforcement and resin materials types and Table 3. for different short reinforcement fiber with different powder reinforcement and resin materials types. From tables shown that the mechanical properties of hyper composite materials increasing with increase of volume fraction reinforcement fiber (mat or short and powder reinforcement) and mechanical properties of composite increase with increase of mat or short fiber more than increasing with increase of powder reinforcement. And the mechanical properties increasing with increase of powder reinforcement, due to increasing the mechanical properties of composite matrix (resin and powder reinforcement). And, from Tables 2 and 3, the mechanical properties of composite materials with short reinforcement fiber more than the mechanical properties of composite materials with mat reinforcement fiber, sine the effect of short fiber more than the

effect of mat fiber. In addition to, the mechanical properties of composite materials non effect with powder reinforcement types and effect with resin types, as shown in Tables 2 and 3.

The theoretical natural frequency results, evaluated by solution of general equation of motion of isotropic hyper composite plate, are comparing with numerical natural frequency results, evaluated by finite elements methods (Ansys program ver. 14), as shown in figures 4 to 7 for different aspect ratio (AR=0.5, 1, and 2), total volume fraction of reinforcement fiber ($\forall_f$ from 30% to 50%), volume fraction of mat or short reinforcement fiber and powder reinforcement ($\forall_{sf}$ or $\forall_{mf}$ from 15% to 30% and $\forall_{pf}$ from 0% to 35%), reinforcement powder materials types (glass and boron), resin materials types (polyester and epoxy), and hyper composite plate types (reinforcement with mat or short fiber). From figures shown that the compare of theoretical results with numerical results given a good agreement, with maximum error about (1.8%) and minimum error about (0.75%).

Figures 8 to 10. Shown contour of natural frequency of hyper composite plate with effect of mat volume fraction reinforcement fiber and powder reinforcement volume fraction of composite plate combined from glass mat reinforcement fiber and glass or boron powder reinforcement with polyester resin materials for different aspect ratio of composite plate (AR=0.5, 1, and 2, respectively). And, figures 11 to 13 shown the natural frequency of composite plate with effect of volume fraction mat and powder reinforcement of composite plate combined from glass mat reinforcement fiber and glass or boron powder reinforcement with epoxy resin materials for different aspect ratio of plate. From figures shown that the natural frequency for isotropic hyper composite plate increasing with increased volume fraction of reinforcement fiber (mat fiber and powder reinforcement), due to increasing the strength of composite plate with increasing volume fraction of reinforcement in composite materials plate.

And, from figures 8 to 13, the natural frequency of composite plate increase with increasing volume fraction of mat fiber more than increasing of natural frequency with increase of powder reinforcement, since the strength of composite materials increase with mat reinforcement more than strength of composite materials with powder reinforcement. In addition to, the natural frequency of composite plate combined from reinforcement and epoxy resin materials more than the natural frequency of composite plate combined from reinforcement and polyester resin materials, since the strength of epoxy more than strength of polyester materials. And, reinforcement with powder causes increases the natural frequency of composite plate due to increasing strength of matrix. And, the natural frequency of plate decrease with increase aspect ratio of plate due to decreasing of strength to weight ratio of composite plate.

Figures 14 to 16. Shown contour of natural frequency of isotropic hyper composite plate with effect of short reinforcement fiber volume fraction and powder volume fraction of composite plate combined from glass short reinforcement fiber and glass or boron powder reinforcement with polyester resin materials for different aspect ratio of composite plate (AR=0.5, 1, and 2, respectively). And, figures 17 to 19 shown the natural frequency of composite plate with effect of volume fraction of reinforcement fiber (short and powder reinforcement) of composite plate combined from glass mat reinforcement fiber and glass or boron powder reinforcement materials with epoxy resin materials for different aspect ratio (AR=0.5, 1, and 2, respectively). Also, the effect of short reinforcement fiber on the natural frequency behaviour of hyper composite plate as the effect of mat reinforcement fiber on behaviour of the natural frequency of isotropic hyper composite plate structural showed in the figures 8 to 13.

From figures 8 to 13, for mat reinforcement fiber, and figures 14 to 19, for short reinforcement fiber, shown the natural frequency of composite plate had reinforcement with short reinforcement fiber more than the natural frequency of composite plate had reinforcement with mat reinforcement fiber, since the strength of composite materials plate

reinforcement with short fiber more than strength of composite materials plate reinforcement with mat fiber (for same density of mat and short reinforcement fiber), as shown in the Tables 2 and 3.

Figures 20 and 21. shown the natural frequency of isotropic hyper composite plate combined from, glass mat reinforcement fiber, glass or boron reinforcement powder, and polyester and epoxy resin matrix materials, respectively, with various mat volume fraction reinforcement fiber effect, with different aspect ratio (AR=0.5, 1, and 2). From figures shown that the natural frequency of plate increasing with decrease aspect ratio of plate, due to decrease volume of plate structural (decrease the weight of composite plate, with same strength of plate). And the natural frequency of composite plate had epoxy resin more than the natural frequency of composite plate had polyester resin materials, since the strength of epoxy more than strength of polyester material resin.

And figures 22 and 23 shown same effect of aspect ratio and resin materials types for composite plate combined from glass short reinforcement fiber, boron or glass powder reinforcement, and polyester and epoxy resin materials, respectively. From figures 20 to 23 shown the natural frequency of plate reinforcement with short fiber more than the natural frequencies of composite plate reinforcement with mat reinforcement fiber. And, the natural frequency of composite plate increase with increasing of mat or short reinforcement fiber more than increase of reinforcement powder, since the effect of short or mat reinforcement fiber on the strength of composite materials more than the effect of reinforcement powder on strength of composite materials plate.

Figures 24 and 25 shown the natural frequency of hyper composite plate reinforcement with different volume fraction of glass mat fiber with glass and boron reinforcement powder with polyester and epoxy resin materials, respectively, and aspect ratio of plate is 0.5 and total volume fraction of reinforcement, mat fiber and powder (50%). And figures 26 and 27 showed the natural frequency of composite plate reinforcement with glass short reinforcement fiber. From figures shown the strength of reinforcement powder non-effect on the natural frequency of composite plate, the reinforcement powder types non-effect on natural frequency of plate (with same density of reinforcement powder), since the mechanical properties of powder materials non effect on the mechanical properties of composite materials. Then, the natural frequencies of composite plate reinforcement with high density powder less than natural frequency of composite plate reinforcement with low density powder materials, since the strength of powder non-effect on natural frequency plate.

Figures 28 and 29 shown the effect of resin materials types, polyester and epoxy resin materials, on the natural frequency of hyper composite plate with different reinforcement fiber types (glass mat and short reinforcement, respectively). From the figures show the natural frequency of plate with epoxy resin materials more than natural frequency of plate with polyester resin materials, since the strength of epoxy greater than strength of polyester materials, then the strength of composite materials with epoxy resin more than strength of composite materials with polyester resin.

Figures 30 to 32 shown the effect of reinforcement fiber types (mat and short reinforcement fiber types) on the natural frequency of different aspect ratio composite plate (AR=0.5, 1, and 2, respectively) with polyester resin materials. And, figures 33 to 35 show the natural frequency of composite plate with effect of fiber type (mat and short reinforcement fiber types) and aspect ratio of plate (AR=0.5, 1, and 2, respectively) for epoxy resin materials. since, the effect of short fiber on mechanical properties of composite materials more than the effect of mat reinforcement fiber, then, the natural frequency with short fiber more than natural frequency of hyper composite plate with mat reinforcement fiber.

**Table 2: Mechanical Properties of Isotropic Hyper Composite Plate Combined of Glass Mat Reinforcement Fiber and Different Reinforcement Powder Materials Types and Different Resin Matrix Materials Types**

| Volume Fraction of Resin | Volume Fraction of Reinforcement | Volume Fraction of Mat Fiber | Volume Fraction of Powder Fiber | Mat Glass Fiber and Glass or Boron Powder Reinforcement and Polyester Resin | | | Mat Glass Fiber and Glass or Boron Powder Reinforcement and Epoxy Resin | | | ρ ($kg/m^3$) |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | E (Gpa) | G (Gpa) | ν | E (Gpa) | G (Gpa) | ν | |
| 70 % | 30 % | 15 % | 15 % | 10.285 | 3.956 | 0.3 | 11.042 | 4.247 | 0.3 | 1620 |
| | | 20 % | 10 % | 11.15 | 4.288 | 0.3 | 11.84 | 4.554 | 0.3 | 1620 |
| | | 25 % | 5 % | 12.012 | 4.62 | 0.3 | 12.636 | 4.86 | 0.3 | 1620 |
| | | 30 % | 0 % | 12.866 | 4.948 | 0.3 | 13.422 | 5.162 | 0.3 | 1620 |
| 65 % | 35 % | 15 % | 20 % | 11.121 | 4.277 | 0.3 | 11.98 | 4.608 | 0.3 | 1690 |
| | | 20 % | 15 % | 11.953 | 4.597 | 0.3 | 12.741 | 4.9 | 0.3 | 1690 |
| | | 25 % | 10 % | 12.785 | 4.917 | 0.3 | 13.502 | 5.193 | 0.3 | 1690 |
| | | 30 % | 5 % | 13.612 | 5.235 | 0.3 | 14.258 | 5.484 | 0.3 | 1690 |
| 60 % | 40 % | 15 % | 25 % | 12.091 | 4.651 | 0.3 | 13.069 | 5.026 | 0.3 | 1760 |
| | | 20 % | 20 % | 12.884 | 4.955 | 0.3 | 13.784 | 5.302 | 0.3 | 1760 |
| | | 25 % | 15 % | 13.681 | 5.262 | 0.3 | 14.505 | 5.579 | 0.3 | 1760 |
| | | 30 % | 10 % | 14.475 | 5.567 | 0.3 | 15.223 | 5.855 | 0.3 | 1760 |
| 55 % | 45 % | 15 % | 30 % | 13.233 | 5.089 | 0.3 | 14.348 | 5.519 | 0.3 | 1830 |
| | | 20 % | 25 % | 13.978 | 5.376 | 0.3 | 15.009 | 5.773 | 0.3 | 1830 |
| | | 25 % | 20 % | 14.73 | 5.665 | 0.3 | 15.679 | 6.03 | 0.3 | 1830 |
| | | 30 % | 15 % | 15.484 | 5.955 | 0.3 | 16.352 | 6.289 | 0.3 | 1830 |
| 50 % | 50 % | 15 % | 35 % | 14.595 | 5.613 | 0.3 | 15.875 | 6.106 | 0.3 | 1900 |
| | | 20 % | 30 % | 15.281 | 5.877 | 0.3 | 16.467 | 6.334 | 0.3 | 1900 |
| | | 25 % | 25 % | 15.978 | 6.145 | 0.3 | 17.074 | 6.567 | 0.3 | 1900 |
| | | 30 % | 20 % | 16.682 | 6.416 | 0.3 | 17.69 | 6.804 | 0.3 | 1900 |

**Table 3: Mechanical Properties of Isotropic Hyper Composite Plate Combined of Glass Short Reinforcement Fiber and Different Reinforcement Powder Materials Types and Different Resin Matrix Materials Types**

| Volume Fraction of Resin | Volume Fraction of Reinforcement | Volume Fraction of Short Fiber | Volume Fraction of Powder Fiber | Short Glass Fiber and Glass or Boron Powder Reinforcement and Polyester Resin | | | Short Glass Fiber and Glass or Boron Powder Reinforcement and Epoxy Resin | | | ρ ($kg/m^3$) |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | E (Gpa) | G (Gpa) | ν | E (Gpa) | G (Gpa) | ν | |
| 70 % | 30 % | 15 % | 15 % | 11.007 | 4.013 | 0.372 | 11.855 | 4.335 | 0.367 | 1620 |
| | | 20 % | 10 % | 12.058 | 4.365 | 0.381 | 12.871 | 4.676 | 0.376 | 1620 |
| | | 25 % | 5 % | 13.064 | 4.699 | 0.39 | 13.844 | 4.998 | 0.385 | 1620 |
| | | 30 % | 0 % | 14.005 | 5.007 | 0.399 | 14.75 | 5.293 | 0.393 | 1620 |
| 65 % | 35 % | 15 % | 20 % | 11.943 | 4.368 | 0.367 | 12.891 | 4.728 | 0.363 | 1690 |
| | | 20 % | 15 % | 13.003 | 4.726 | 0.376 | 13.911 | 5.072 | 0.371 | 1690 |
| | | 25 % | 10 % | 14.028 | 5.068 | 0.384 | 14.898 | 5.401 | 0.379 | 1690 |
| | | 30 % | 5 % | 15 | 5.389 | 0.392 | 15.833 | 5.709 | 0.387 | 1690 |
| 60 % | 40 % | 15 % | 25 % | 13.012 | 4.774 | 0.363 | 14.074 | 5.178 | 0.359 | 1760 |
| | | 20 % | 20 % | 14.075 | 5.135 | 0.371 | 15.09 | 5.522 | 0.366 | 1760 |
| | | 25 % | 15 % | 15.112 | 5.483 | 0.378 | 16.084 | 5.855 | 0.373 | 1760 |
| | | 30 % | 10 % | 16.107 | 5.814 | 0.385 | 17.038 | 6.172 | 0.38 | 1760 |
| 55 % | 45 % | 15 % | 30 % | 14.25 | 5.245 | 0.359 | 15.443 | 5.698 | 0.355 | 1830 |
| | | 20 % | 25 % | 15.306 | 5.604 | 0.366 | 16.445 | 6.038 | 0.362 | 1830 |
| | | 25 % | 20 % | 16.346 | 5.956 | 0.372 | 17.434 | 6.372 | 0.368 | 1830 |
| | | 30 % | 15 % | 17.356 | 6.294 | 0.379 | 18.397 | 6.694 | 0.374 | 1830 |
| 50 % | 50 % | 15 % | 35 % | 15.705 | 5.797 | 0.355 | 17.052 | 6.308 | 0.352 | 1900 |
| | | 20 % | 30 % | 16.741 | 6.151 | 0.361 | 18.023 | 6.64 | 0.357 | 1900 |
| | | 25 % | 25 % | 17.773 | 6.502 | 0.367 | 18.994 | 6.969 | 0.363 | 1900 |
| | | 30 % | 20 % | 18.786 | 6.844 | 0.373 | 19.952 | 7.292 | 0.368 | 1900 |

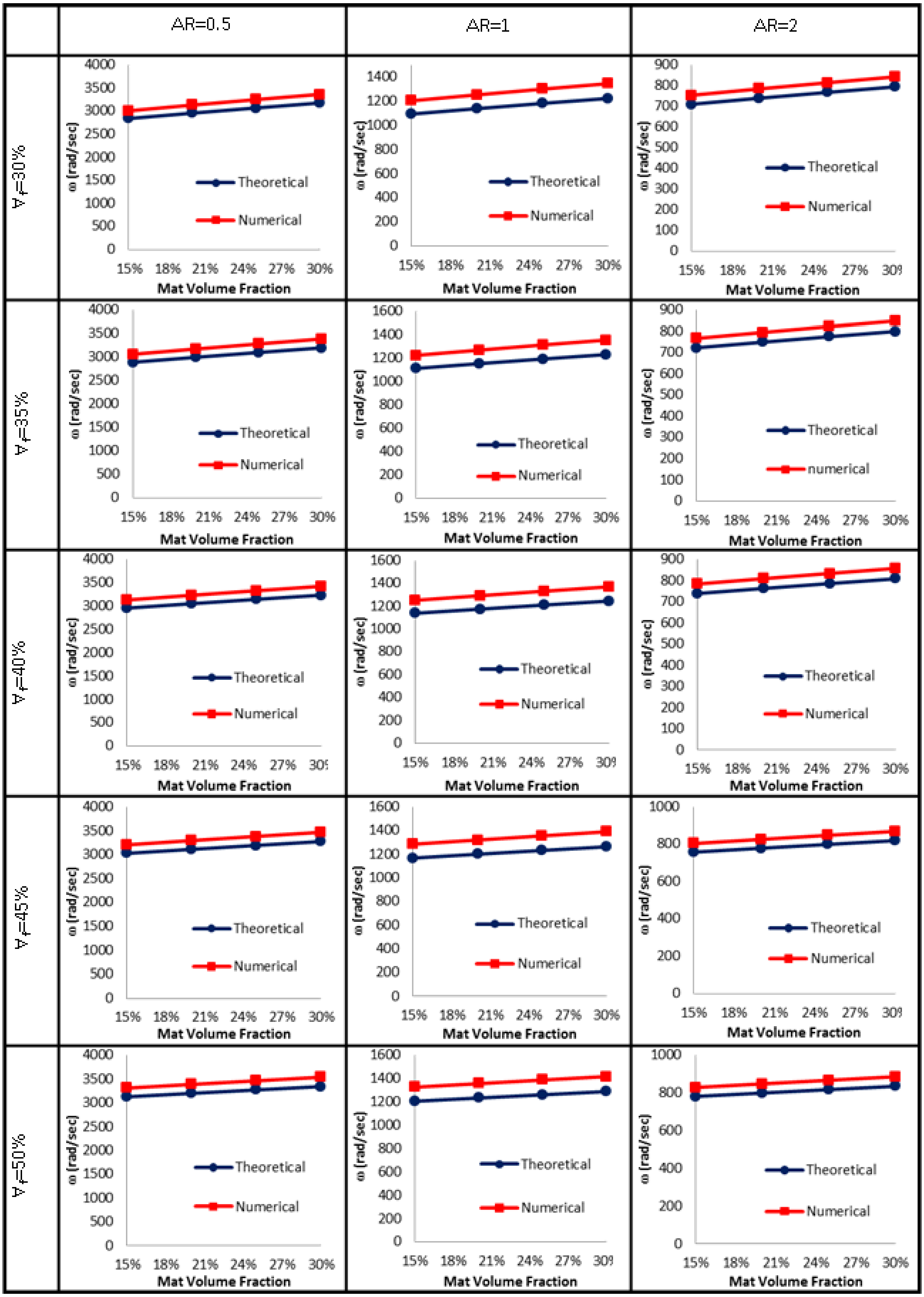



**Figure 4: Compare between Theoretical and Numerical Study for Glass Mat Fiber, with Glass or Boron Powder and Polyester Resin, for Different Aspect Ratio and Volume Fraction Fiber**

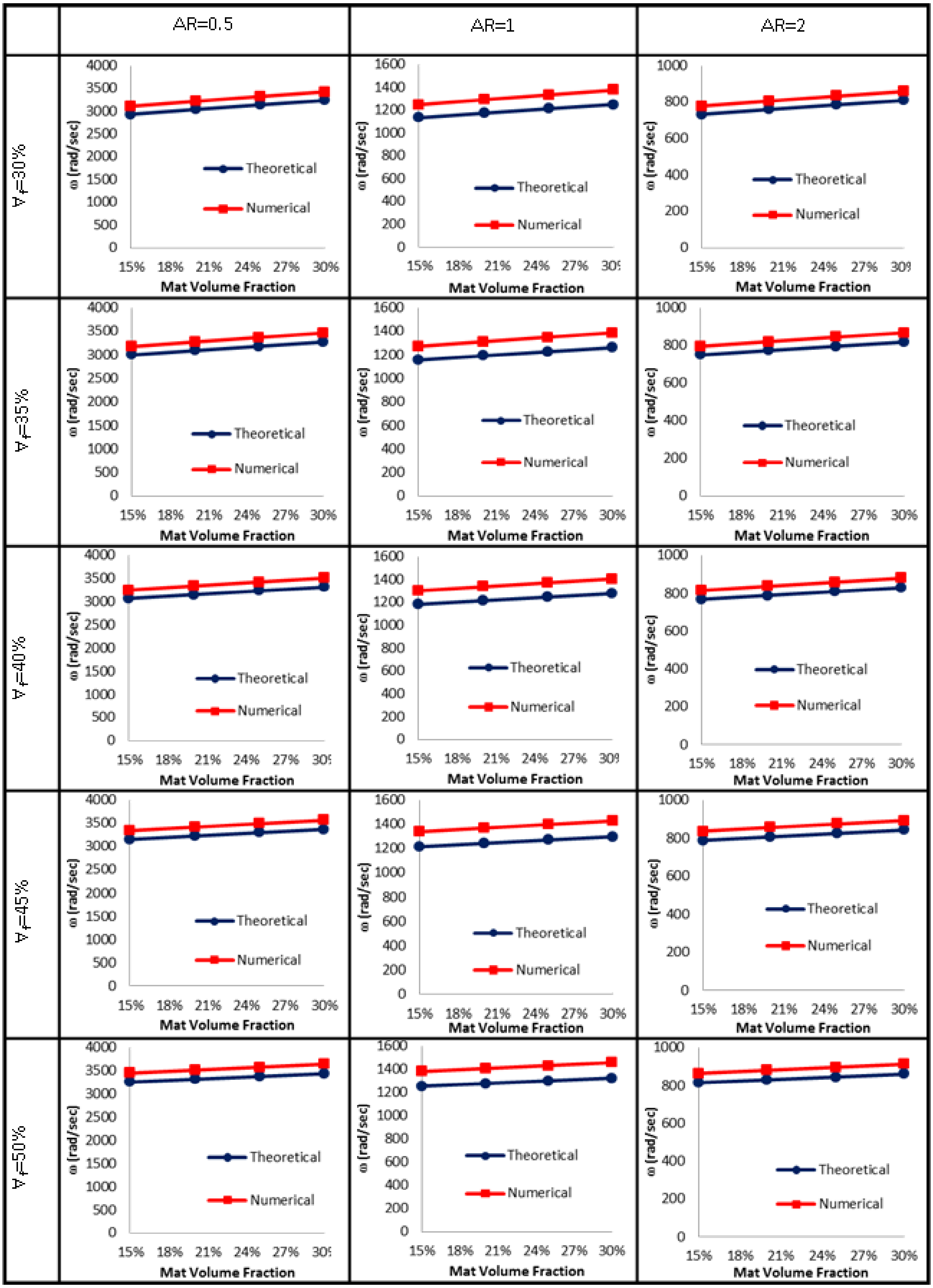



**Figure 5: Compare between Theoretical and Numerical Study for Glass Mat Fiber, with Glass or Boron Powder and Epoxy Resin, for Different Aspect Ratio and Volume Fraction Fiber**

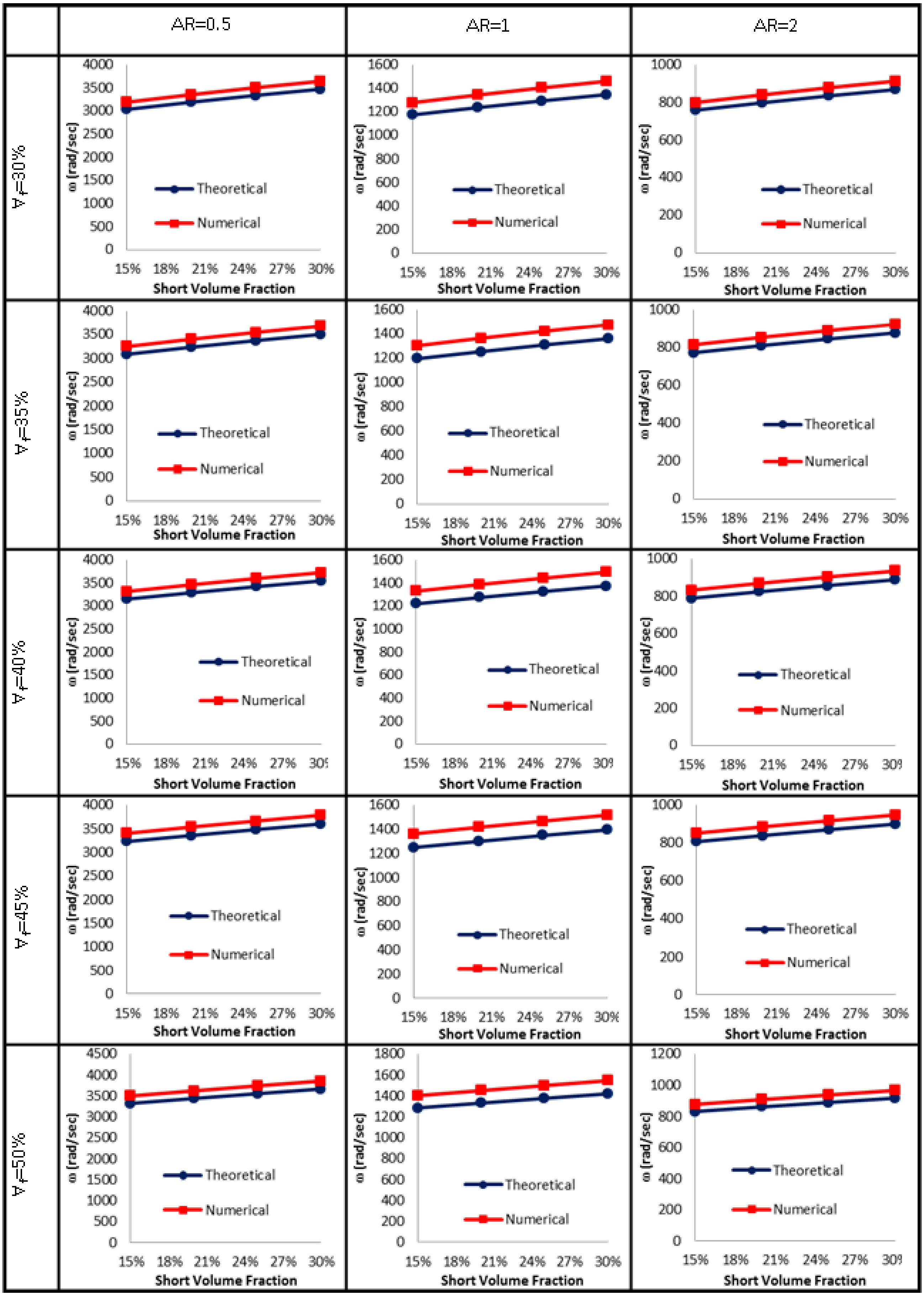



**Figure 6: Compare between Theoretical and Numerical Study for Glass Short Fiber, with Glass or Boron Powder and Polyester Resin, for Different Aspect Ratio and Volume Fraction Fiber**

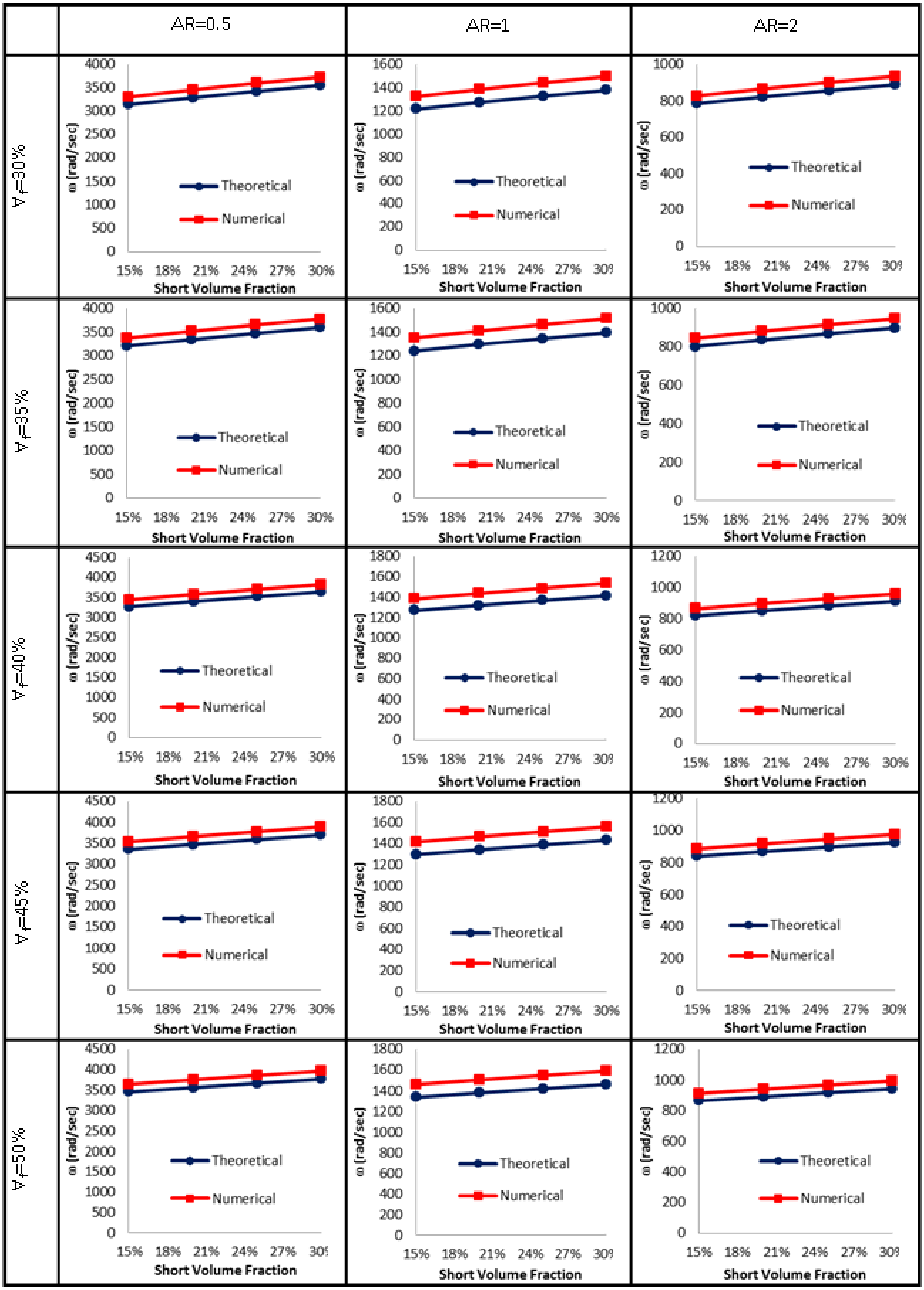



**Figure 7: Compare between Theoretical and Numerical Study for Glass Short Fiber, with Glass or Boron Powder and Epoxy Resin, for Different Aspect Ratio and Volume Fraction Fiber**

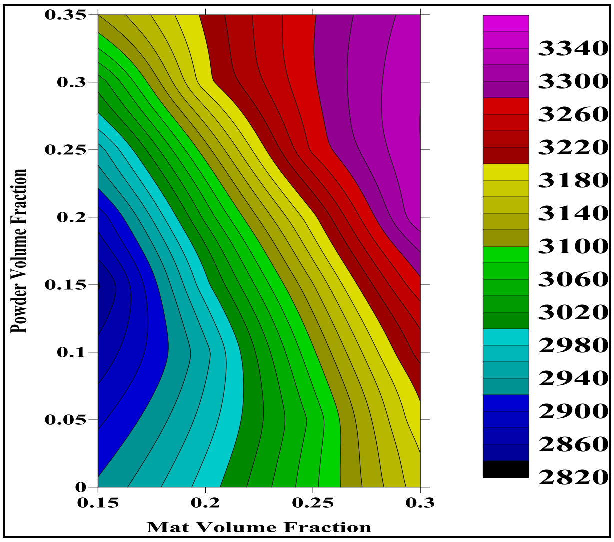



Figure 8: Natural Frequency of Mat Fiber Hyper Composite Plate with Polyester Resin and AR=0.5

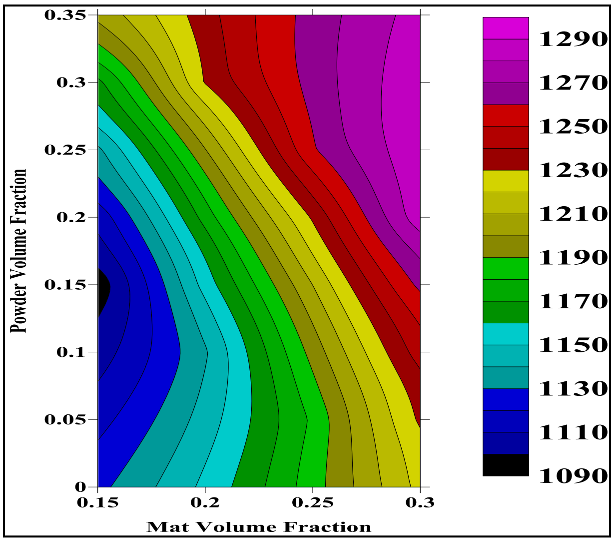



Figure 9: Natural Frequency of Mat Fiber Hyper Composite Plate with Polyester Resin and AR=1

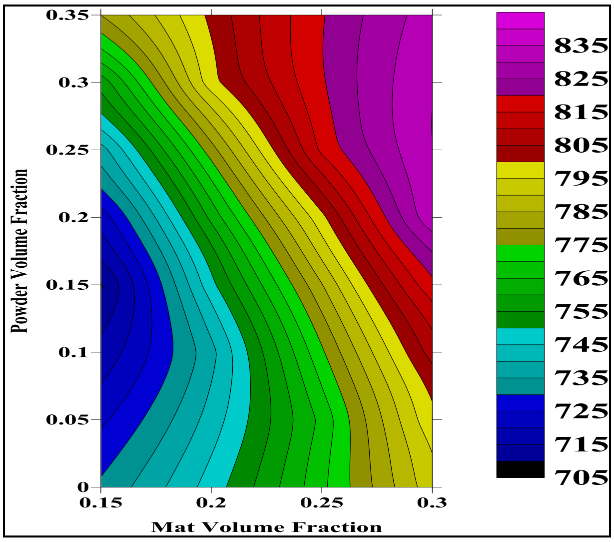



Figure 10: Natural Frequency of Mat Fiber Hyper Composite Plate with Polyester Resin and AR=2

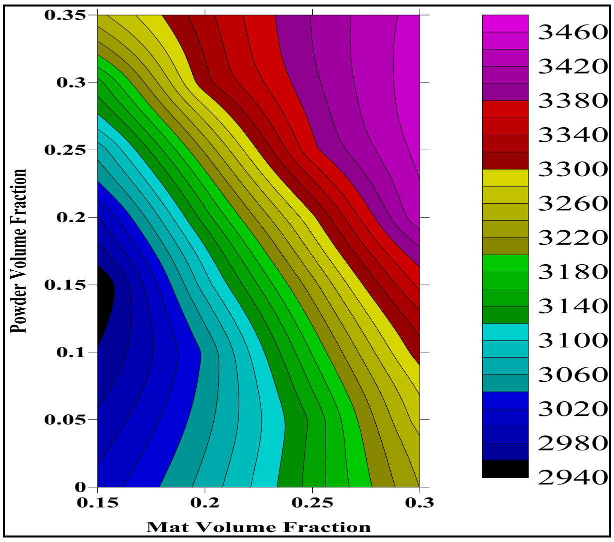



Figure 11: Natural Frequency of Mat Fiber Hyper Composite Plate with Epoxy Resin and AR=0.5

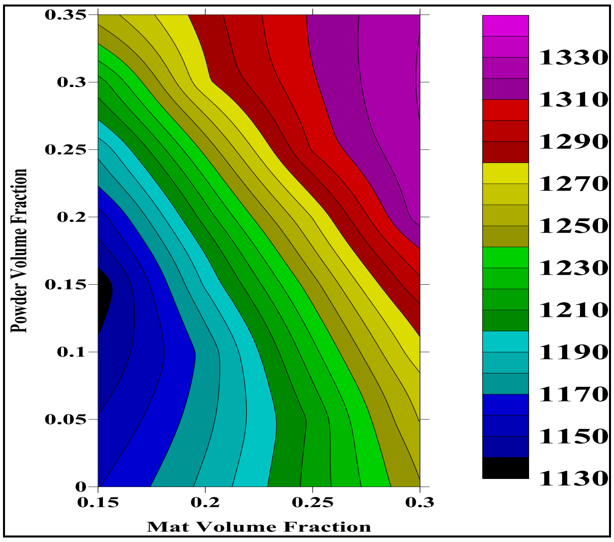



Figure 12: Natural Frequency of Mat Fiber Hyper Composite Plate with Epoxy Resin and AR=1

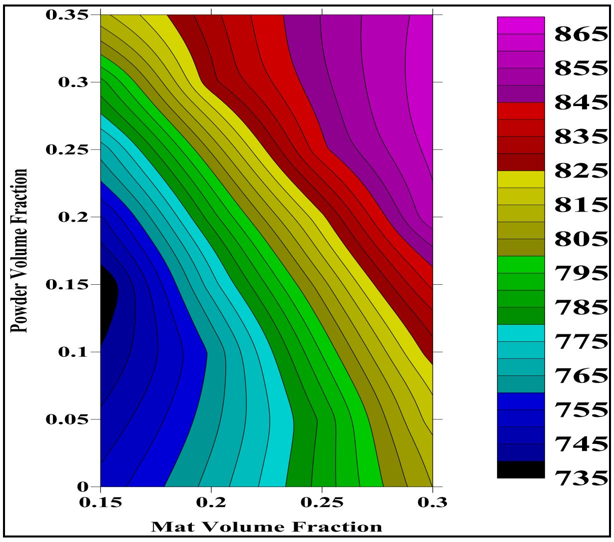



Figure 13: Natural Frequency of Mat Fiber Hyper Composite Plate with Epoxy Resin and AR=2

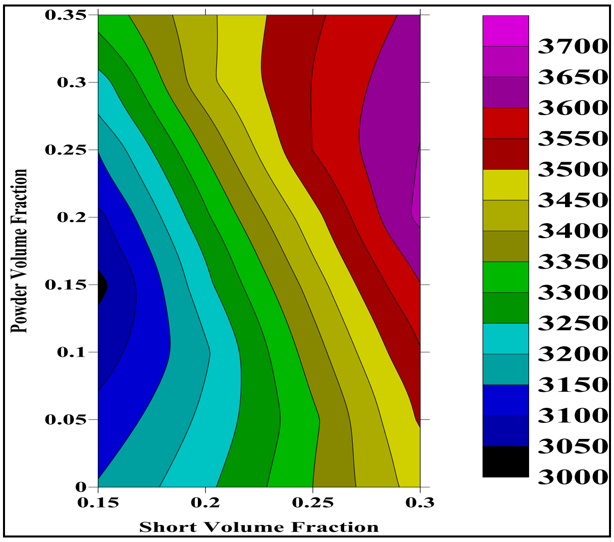



**Figure 14: Natural Frequency of Short Fiber Hyper Composite Plate with Polyester Resin and AR=0.5**

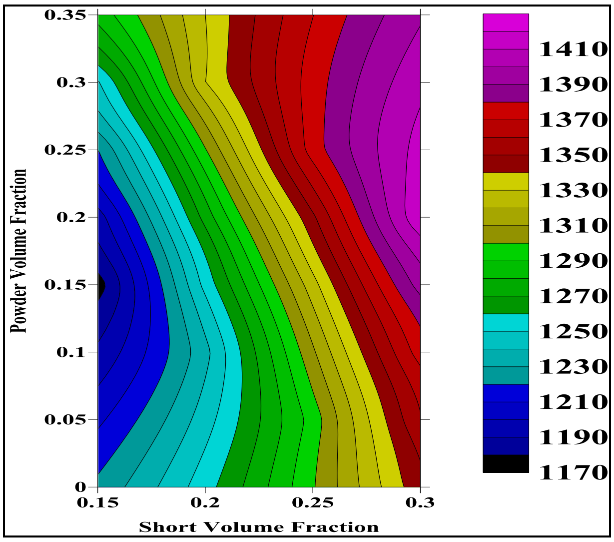



**Figure 15: Natural Frequency of Short Fiber Hyper Composite Plate with Polyester Resin and AR=1**

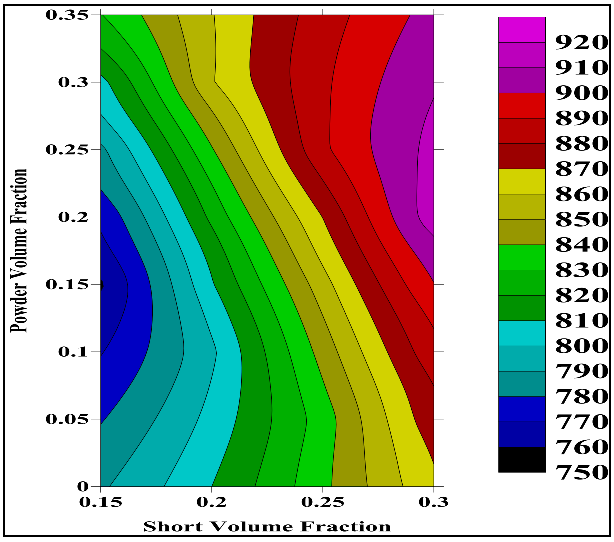



**Figure 16: Natural Frequency of Short Fiber Hyper Composite Plate with Polyester Resin and AR=2**

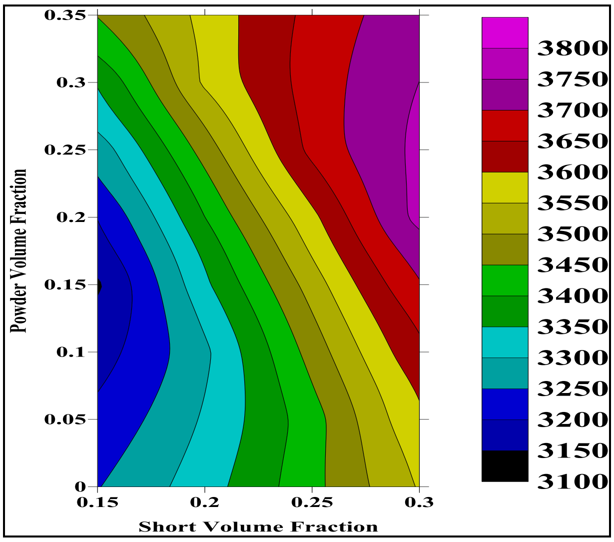



**Figure 17: Natural Frequency of Short Fiber Hyper Composite Plate with Epoxy Resin and AR=0.5**

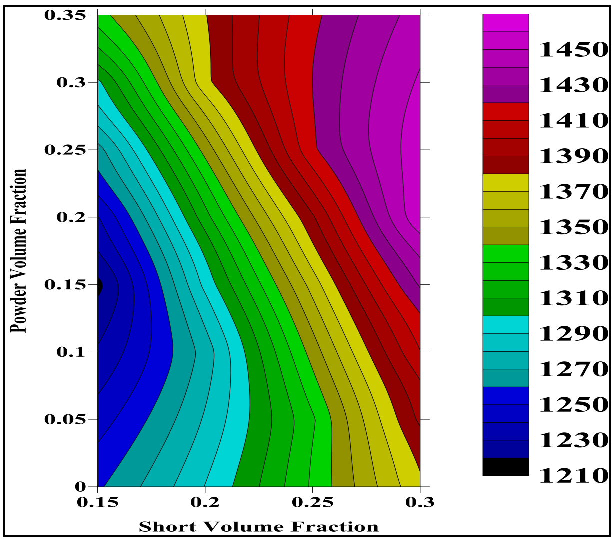



**Figure 18: Natural Frequency of Short Fiber Hyper Composite Plate with Epoxy Resin and AR=1**

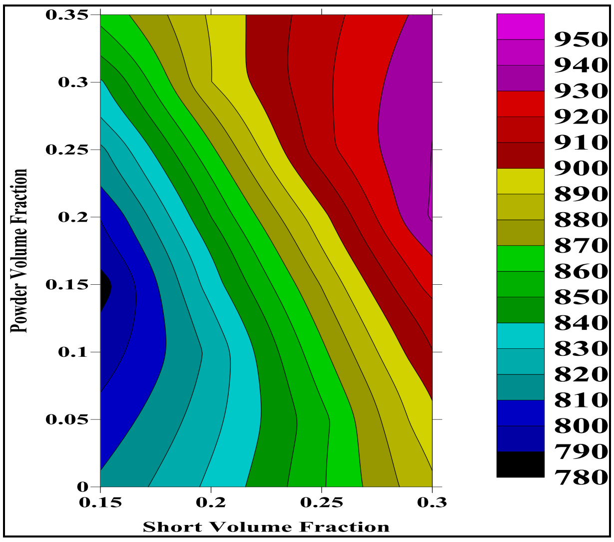



**Figure 19: Natural Frequency of Short Fiber Hyper Composite Plate with Epoxy Resin and AR=2**

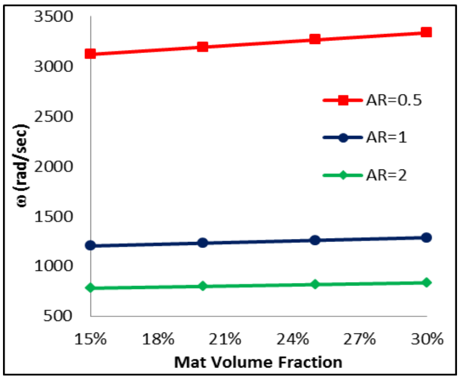



Figure 20: Natural Frequency of Mat Fiber Hyper Composite Plate with Polyester Resin and $\forall_f=50\%$

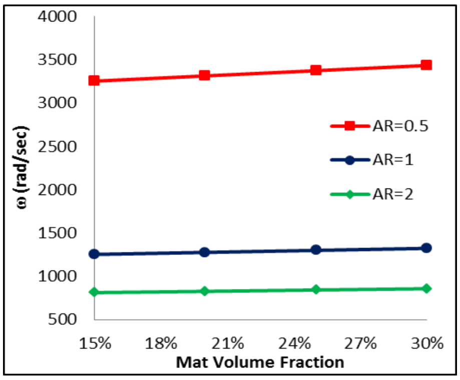



Figure 21: Natural Frequency of Mat Fiber Hyper Composite Plate with Epoxy Resin and $\forall_f=50\%$

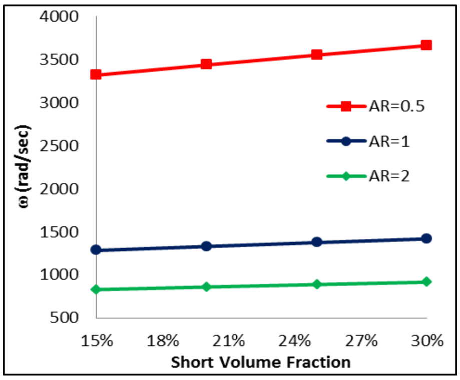



Figure 22: Natural Frequency of Short Fiber Hyper Composite Plate with Polyester Resin and $\forall_f=50\%$

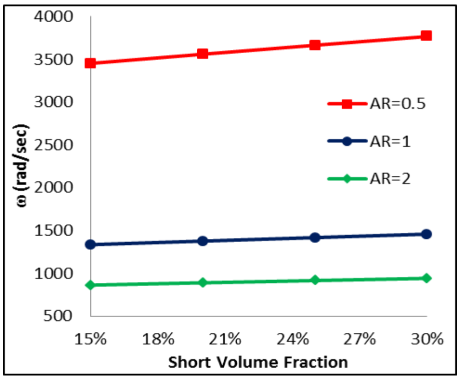



Figure 23: Natural Frequency of Short Fiber Hyper Composite Plate with Epoxy Resin and $\forall_f=50\%$

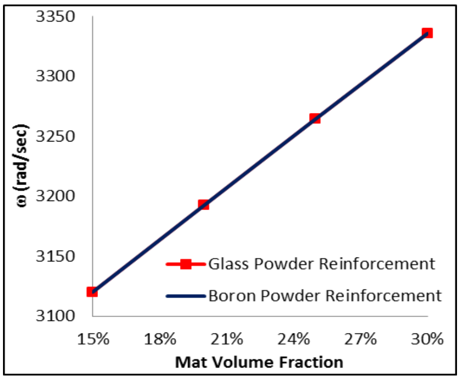



Figure 24: Natural Frequency of Mat Fiber Hyper Plate with Polyester Resin and $\forall_f=50\%$, AR=0.5

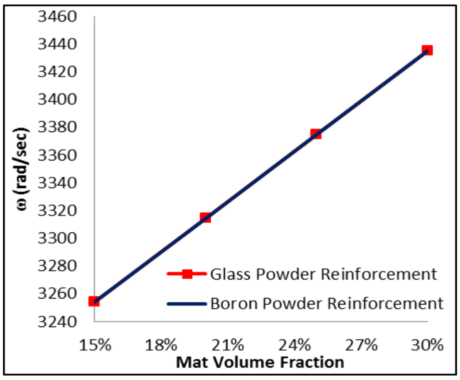



Figure 25: Natural Frequency of Mat Fiber Hyper Plate with Epoxy Resin and $\forall_f=50\%$, AR=0.5

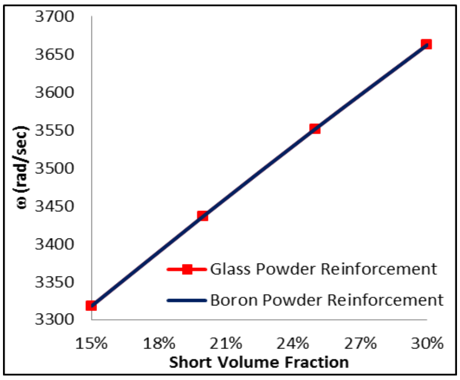



Figure 26: Natural Frequency of Short Fiber Hyper Plate with Polyester Resin and $\forall_f=50\%$, AR=0.5

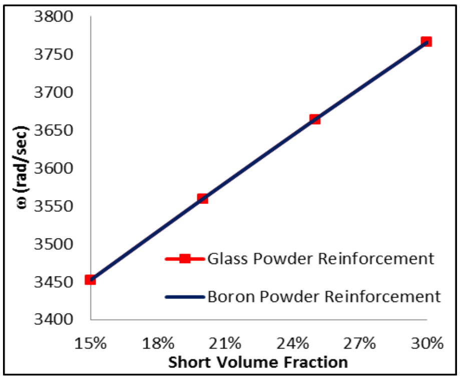



Figure 27: Natural Frequency of Short Fiber Hyper Plate with Epoxy Resin and $\forall_f=50\%$, AR=0.5

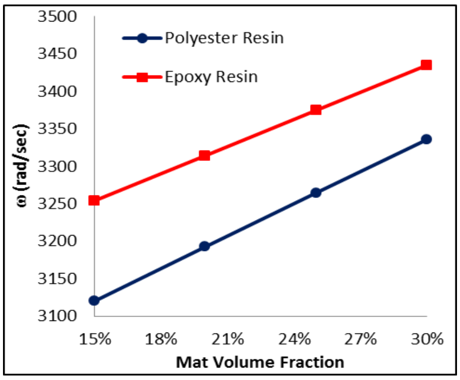



**Figure 28: Natural Frequency of Glass Mat Fiber with Different Resin and $\forall_f$=50%, AR=0.5**

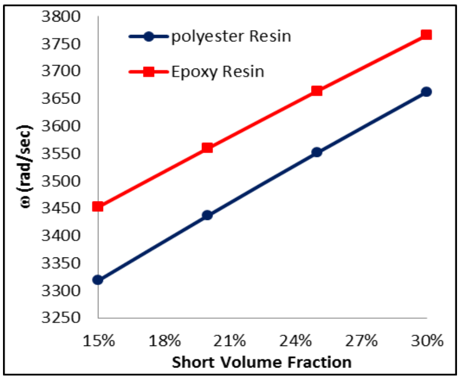



**Figure 29: Natural Frequency of Glass Short Fiber with Different Resin and $\forall_f$=50%, AR=0.5**

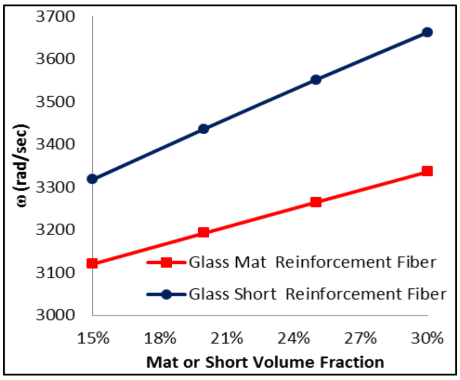



**Figure 30: Natural Frequency of Different Fiber with Polyester Resin and $\forall_f$=50%, AR=0.5**

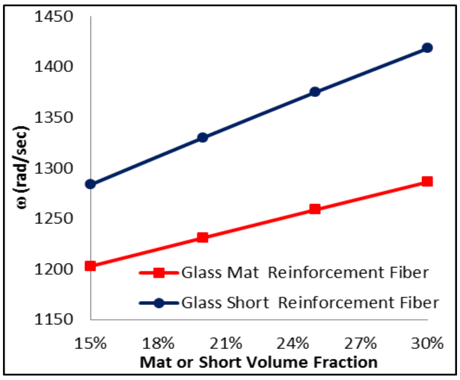



**Figure 31: Natural Frequency of Different Fiber with Polyester Resin and $\forall_f$=50%, AR=1**

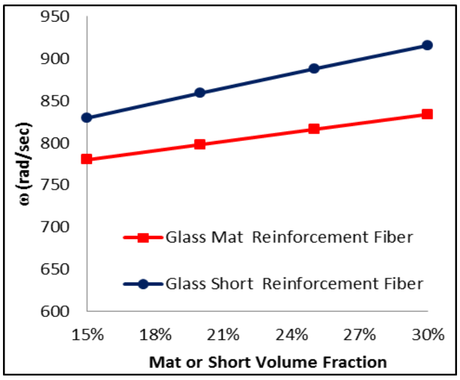



**Figure 32: Natural Frequency of Different Fiber with Polyester Resin and $\forall_f$=50%, AR=2**

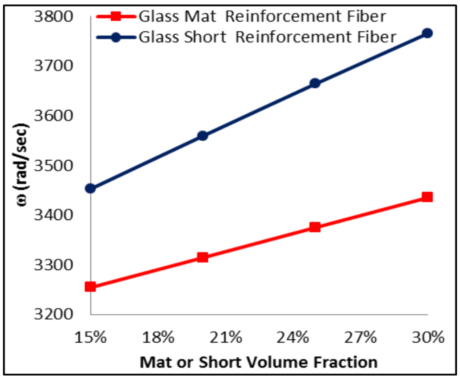



**Figure 33: Natural Frequency of Different Fiber with Epoxy Resin and $\forall_f$=50%, AR=0.5**

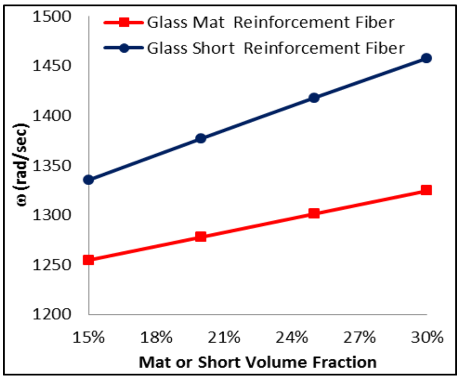



**Figure 34: Natural Frequency of Different Fiber with Epoxy Resin and ∀f=50%, AR=1**

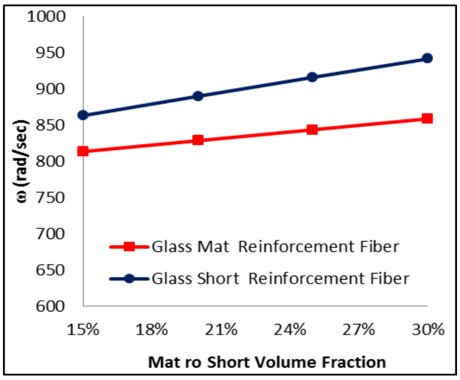



**Figure 35: Natural Frequency of Different Fiber with Epoxy Resin and ∀f=50%, AR=2**

## CONCLUSIONS

Some concluding observations from the investigation are given below:

- The suggested analytical solution is a powerful tool for vibration analysis study of hyper isotropic composite plate with different volume fraction of reinforcement and resin materials, by solution the general differential equations of motion of isotropic plated by using separation and orthogonal methods for differential equation of vibration plate.
- The natural frequency and strength of hyper composite plate increase with reinforcement by powder fiber, but types of powder materials fiber non-effect on the natural frequency and strength of hyper composite materials plate, with same density of powder reinforcement. And the natural frequency increasing with increasing strength of resin matrix.
- The natural frequency of hyper composite plate decrease with increasing density of reinforcement powder materials, decreasing of strength to weight ratio of composite materials plate.
- The natural frequency of hyper composite materials plate increase with increasing of reinforcements volume fraction (powder and mat or short fiber) and decrease with increasing of resin matrix volume fraction.
- The natural frequency of hyper composite materials plate increase with increasing of short or mat reinforcement fiber volume fraction more than increasing with increasing of powder reinforcement volume fraction.
- The natural frequency of isotropic hyper composite materials plate with short reinforcement fiber more than the natural frequency of isotropic hyper composite materials plate with mat reinforcement fiber, strength of composite materials with short reinforcement fiber more than strength of composite materials with mat reinforcement fiber.

## REFERENCES

1. Daniel Gay, Suong V. Hoa and Stephen W. Tsai (2003) 'Composite Materials Design and Applications' Book, CRC Press LLC.
2. J. S. Rao (1999) 'Dynamics of Plates' Narosa Publishing House.
3. Muhannad Al-Waily (2005) "Analysis of Stiffened and Un-Stiffened laminated Plates Subjected to Time Dependant Loading" M.Sc. Thesis Submitted to the Mechanical Engineering Department, College of Engineering, Kufa University.
4. T. Kan, K. Swaminathan (2002) ‘Analytical Solutions for the Static Analysis of Laminated Composite and Sandwich Plates Based on a Higher Order Refined Theory’ Composite Structures, Vol. 56, pp. 329-344.
5. R.C.L. Dutra, B.G. Soares, E.A. Campos, J.L.G. Silva (2000) ‘Hybrid Composites Based on Polypropylene and Carbon Fiber and Epoxy Matrix’ Elsevier-Polymer, Vol. 41, pp. 3841–3849.
6. Mohammad A. Wahab, Il Kim, Chang-Sik Ha (2003) ‘Microstructure and Properties of Polyimide/Poly (Vinylsilsesquioxane) Hybrid Composite Films’ Elsevier-Polymer, Vol. 44, pp. 4705–4713.
7. K.K. Shukla, Jhao-Ming Chen, Jin H. Huang (2004) ‘Nonlinear Dynamic Analysis of Composite Laminated Plates Containing Spatially Oriented Short Fibers’ International Journal of Solids and Structures, Vol. 41.
8. Rajesh Kumar and Chandra Shakher (2008) ‘Application of Plate Vibration and DSPI in Evaluation of Elastic Modulus’ Ninth International Symposium on Laser Metrology, edited by Chenggen Quan, Anand Asundi, Proc. of SPIE Vol. 7155, 71550U.

9. Ali Mohammed Hussein Yusif Al-Hajjar (2013) ‘Analytical and Numerical Vibration Study of Unidirectional Hyper Composite Plate’ Asian Transactions on Engineering, Vol. 3, No. 2.
10. Bryan Harris (1999) ‘Engineering Composite Materials’ The Institute of Materials, London.
11. Gururaja M N, A N Hari Rao (2012) ‘A Review on Recent Applications and Future Prospectus of Hybrid Composites’ International Journal of Soft Computing and Engineering, Vol. 1, No. 6.
12. Ali S. Humod and Muhannad Al-Waily (2010) ’ Experimental and Analytical Study of Tensile Properties For Hyper Composite Material’ The Iraqi Journal For Mechanical And Material Engineering, Vol.10, No.1.
13. K. Sabeel Ahmed and S. Vijayarangan (2006) ‘Elastic Property Evaluation of Jute-Glass Fibre Hybrid Composite Using Experimental and CLT Approach’ Indian Journal of Engineering and Materials Sciences, Vol. 13.
14. B. Sutharson, M. Rajendran, S. R. Devadasan and B. Selvam (2012) ‘Effect of Chemical Treatments on Mechanical Properties of Jute Fiber Hybrid Composite Laminates’ ARPN Journal of Engineering and Applied Sciences, Vol. 7, No. 6.
15. Takeshi Takatoya, Yutaka Sato, Ippei Susuki, and Naohito Tsuda (2007) ‘Compression After Impact Properties of Hybrid Composite Materials’ $16^{TH}$ International Conference on Composite Materials.
16. Luay S. Al-Ansari , Muhannad Al-Waily and Ali M. H. Yusif (2012) ‘Vibration Analysis of Hyper Composite Material Beam Utilizing Shear Deformation and Rotary Inertia Effects’ International Journal of Mechanical & Mechatronics Engineering IJMME-IJENS Vol. 12, No. 4.
17. Randall J. Allemang (1999) ‘Vibrations: Analytical and Experimental Modal Analysis’ UC-SDRL.
18. Singiresu S.Rao (1995) ‘Mechanical Vibrations’ Addison-Wesley Publishing Book company.
19. Meirovitch, H. (1997) 'Principles and Techniques of Vibration' prentice-Hall.
20. G. Saheli , H. Garmestani , and B. Adams (2004) ‘Microstructure Design of a Two Phase Composite using Two-Point Correlation Functions’ Journal of Computer-Aided Materials Design.
21. Anil Kumar Dash (2010) ‘Large Amplitude Free Vibration Analysis of Composite Plates by Finite Element Method’ A Thesis of Master of Technology in Structural Engineering, Department of Civil Engineering National Institute of Technology.
22. A. C. Ugural (1999) 'Stress in Plates and Shells, Second Edition' McGraw-Hill Book Companies.
23. Eduard Ventsel and Theodor Krauthammer (2001) ‘Thin Plates and Shells, Theory, Analysis, and Applications’ Book, Marcel Dekker, Inc.
24. Werner Soedel (2004) ‘Vibrations of Shells and Plates, Third Edition, Revised and Expanded’ Book, Marcel Dekker, Inc.
25. Muhannad Al-Waily (2012) "Investigation of Health Monitoring of Composite Plate Structures-Using Vibration Analysis" Ph.D. Thesis Submitted to the Mechanical Engineering Department, College of Engineering, Alnahrain University.
26. S. Timoshenko and S. Woinowsky Krieger (1987) ‘Theory of Plates and Shells’ McGraw-Hill Book Company.